Scalable and accurate quantum tomography from fewer measurements

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Abstract—Quantum tomography is a matrix recovery problem that is not amenable to nuclear-norm minimization. We review existing recovery strategies and propose new ones, with an emphasis on methods with provable guarantees. An outcome of this is a new method with guarantees that is more accurate and more scalable than previous methods.

A single quantum system of \( q \)-qubits \( |\psi\rangle \) can be described by a \( 2^q \times 2^q \) positive semi-definite matrix \( X \) that has trace 1 and rank 1. The exponential growth in the number of qubits is due to entanglement, and this accounts for some of the extraordinary ability of quantum computers, but it also poses challenging problems. Quantum state tomography describes the process of taking measurements of \( X \) in order to estimate the underlying state, and is an essential bread-and-butter tool for building components of a quantum computer. Each measurement is slow to acquire, and the system drifts significantly over a time of 10 hours, thus limiting the possible number of measurements. Since the number of required measurements grows exponentially with \( q \), physicists are constrained to small systems, e.g. 8 qubits [1].

Recently, ideas from matrix completion have been applied to the quantum tomography problem, with provable recovery guarantees [2]. However, that approach fails to take advantage of all prior information. This talk fixes these weaknesses and provides alternative provable methods, and shows enormous practical improvements.

Let \( d = 2^q \) and \( X \in \mathbb{C}^{d \times d} \). From a set of linear measurements \( \mathcal{A}(X)_i = \langle A_i, X \rangle = \text{tr}(A_i^* X) \) for Hermitian \( A_i \), the tomography problem is posed as

\[
\min_{X \succeq 0} \frac{1}{2} \|\mathcal{A}(X) - b\|_2^2 \quad \text{such that} \quad \text{tr}(X) = 1, \quad \text{rank}(X) = r \tag{1}
\]

where \( b = \mathcal{A}(X^*) + z \) are noisy measurements of the true rank \( r \) state \( X^* \). This is solved by an iterative hard-thresholding algorithm that requires an eigenvalue decomposition at each step, and then projects the eigenvalues onto the sparse simplex using the recent algorithm of [3]. Unlike the matrix completion problem, \( \mathcal{A} \) satisfies the RIP and this allows one to show the algorithm converges to a neighborhood of the true state \( X^* \) [5].

The convex approach in [2] considers a nuclear-norm penalty approach [6]. Because the nuclear-norm of a psd matrix is the trace, this conflicts with the trace constraint. One may either solve the convex approach and (a) tune the trace parameter to give a low-rank solution and then normalize the answer to have trace 1, or (b) use the true \( \text{tr}(X) = 1 \) constraint, which may not result in a low-rank answer. Figure 1 shows that the non-convex estimator (1) gives more accurate results because it exploits both the rank and trace constraints simultaneously.

In the non-convex approach, we add Nesterov weightings for acceleration. We also consider non-convex approaches given by the reweighted nuclear norm [7] and by explicit low-rank splittings [8], including AltMinSense [9].

Scalability considerations are different than standard matrix completion because the range of \( \mathcal{A}^* \) is not sparse, so Lanczos methods are less appealing. In order to take advantage of block multiplies, we compare randomized linear algebra techniques [10] and stochastic gradient techniques. Using these randomization tools, we solve (1) on a GPU for a 8 qubit system for extremely fast recovery, and solve it on a cluster using MPI for a 16 qubit system.

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REFERENCES


Fig. 1. Recovering a 8 qubit system with rank 2 and 30 dB SNR, showing the median of 10 runs.