Microwave signal measurements

Introduction

- In non TEM lines, V and I are not defined physically
- In TEM lines, V and I are difficult or impossible to measure
- P is always defined
- f is always defined

Microwave signals are characterized by their power and frequency
Power measurement

• Physical means used:
  – Rectifying the signal
  – Convert microwave power to mechanical force
  – Convert microwave power to heat

Rectifying

• Problems: stability, precision, matching, harmonics. Thus filtering and calibration are necessary
• Advantage: fast
Mechanical measurement

- Problems: very weak force, total reflection of the signal

\[ F = \frac{2P \lambda}{c_0 \lambda g} \]

Thermal conversion

- The microwave power is converted to heat (use of absorbers or thermistors)
- The variation of temperature is measured
  - calorimeters (precision measurements)
  - Electronical measurement (usual measurements)
Thermocouples

- A voltage $U$ exists at each junction between two different conductive materials (electrochemical potential)
- This voltage is temperature dependant
- Thus the "hot" junction is compared to the temperature of a reference "cold" junction

$$U \sim T_1 - T_0 \sim P$$
Water calorimeter

- The flow must be constant
- High precision measurements for standards
- Bulky setups, and slow measurements

\[ P = qC_p \Delta T \]

Milliwattmeter using a wheatstone bridge

- The equilibrium is sought for until having \( i=0 \) in the \( \mu \)Ampmeter
- Without the \( \mu \)wave signal (calibration)
- With the \( \mu \)wave signal (measurement)
Milliwattmeter using a wheatstone bridge

- At the equilibrium, without μ-wave signal

\[ P_0 = \frac{U_0^2}{4R} \]

- In presence of the μ-wave signal, the voltage is reduced to reach the equilibrium again.

\[ P_1 = \frac{(U_o - \Delta U)^2}{4R} \]

Thus, the μ-wave power is equal to

\[ P = P_0 - P_1 = \frac{\Delta U(2U_o - \Delta U)}{4R} \]

Thermosensible elements: Bolometers or thermistors
Bolometer

- Thin wire conductor
- When P increases, T and R increase
- Response time: < 1ms

Thermistor

- Resistor with negative temperature coefficient
- Response time: 0.5-1 s

R ↗ when T ↗
R ↖ quand T ↖
Sources of measurement errors

- mismatch
- Thermal drift
- Substitution errors
- $\mu$-wave losses
- Thermoelectrical effect
- Precision of the powermeter
- Pairing of the sensitive elements

Thermal drift

Compensation: measure at the same time the reference and the signal (paired sensitive elements)
Substitution losses

Substitution efficiency

$$\eta_s = \frac{P_o - P_l}{P_{\mu W}} = \frac{\text{Continuous power}}{\text{microwave power}}$$
Pairing of elements

In coaxial systems, we need to avoid that the continuous signal flows into the transmission line: use of paired elements and decoupling capacitors.

RF losses
Mismatch errors: flow chart

\[ P_a = |d|^2 \left( 1 - |\rho_G|^2 \right) \]

Power that would be absorbed by a matched power meter

\[ P_m = \frac{|d|^2 \left( 1 - |\rho_G|^2 \right) \left( 1 - |\rho_L|^2 \right)}{\left| 1 - \rho_G \rho_L e^{-2 j \beta l} \right|^2} \]

Power absorbed by the unmatched power meter

Mismatch errors

We define the mismatch efficiency as

\[ \frac{P_m}{P_a} = \frac{\left( 1 - |\rho_L|^2 \right)}{\left| 1 - \rho_G \rho_L e^{-2 j \beta l} \right|^2} = \eta_d \]

Thus

\[ \left( \frac{1 - |\rho_L|^2}{1 + |\rho_G \rho_L|} \right)^2 \leq \eta_d \leq \frac{\left( 1 - |\rho_L|^2 \right)}{\left( 1 - |\rho_G \rho_L| \right)^2} \]
Frequency measurements

- Geometrical measurements (wavelength, cavity resonance)
- Electronic measurement
Slotted line
Slotted line
Slotted line

\[ U_i = \sqrt{Z_{ci}} \left( a_i + b_i \right) = \sqrt{Z_{ci}} \left( a_i + s_{ii} a_i \right) \]

\[ U_i(z) = \sqrt{Z_{ci}} a_i(z) \left[ \frac{1 + s_{ii} e^{2 j \beta z}}{1 + s_{ii} e^{2 j \beta z} + s_{ii}^2 \sin^2 (\varphi + 2 \beta z)} \right] \]

\[ U_i(z) = \sqrt{Z_{ci}} a_i(z) \left[ 1 + \left| s_{ii} \right|^2 + 2 \left| s_{ii} \right| \cos (\varphi + 2 \beta z) \right] \]
Extrema of $|U_i|$:

\[
U_{\text{max}} = \sqrt{Z_{ci} a_i \left(1 + s_{ii}\right)} \quad \text{en} \quad \varphi + 2 \beta z_i = 2n\pi \\
U_{\text{min}} = \sqrt{Z_{ci} a_i \left(1 - s_{ii}\right)} \quad \text{en} \quad \varphi + 2 \beta z_i = (2n+1)\pi
\]

\[
ROS = VSWR = \frac{U_{\text{max}}}{U_{\text{min}}} = \frac{1 + |s_{ii}|}{1 - |s_{ii}|}
\]

slotted line

- Measure position of two minima in the slotted line. The guided wavelength is given by twice this distance.

- In the case of a rectangular waveguide of section $a \times b$ in the dominant mode, we have

- Where $\lambda_{10}$ is the measured guided wavelength:

\[
\lambda = \frac{\lambda_{10}}{\sqrt{1 + \left(\frac{\lambda_{10}}{2a}\right)^2}}
\]

\[
f = \frac{c_o}{\lambda}
\]
Wavemeter

Wavemeter: principle

source

waveguide
detector

Amplitude of received signal

Length of the cavity
Wavemeter: different layouts

Absorption
source \rightarrow \text{cavity} \rightarrow \text{detector}

Transmission
source \rightarrow \text{cavity} \rightarrow \text{detector}

Wavemeter: example
Type of wavemeters

- Piston annulaire mobile
- Boucle de couplage
  - En ligne coaxiale (mode TEM)
  - En guide d'ondes circulaire (mode TE_{011})

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TE Cavity modes
TM Cavity modes

resonance of a cylindrical cavity

\[ f_{mn} = \sqrt{\frac{c_0}{2\pi} \left( \frac{x_{mn}}{a} \right)^2 + \left( \frac{l\pi}{d} \right)^2} \]

\[ x_{mn} \text{ is the } n_{th} \text{ zero of } J_m (\text{TE}_mn \text{ mode}) \]

\[ x_{mn} \text{ is the } n_{th} \text{ zero of } J_m (\text{TM}_mn \text{ mode}) \]
zero of Jm (Bessel function of the first kind)

<table>
<thead>
<tr>
<th>n</th>
<th>$x_{n1}$</th>
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zero of $J^\prime m$ (derivative of the Bessel function of first kind)

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<tr>
<th>$n$</th>
<th>$p^\prime n1$</th>
<th>$p^\prime n2$</th>
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Resonance frequency in cylindrical cavities

$af$ [GHz-mm]
Frequency counters

- Periods are counted during a specified time
- A high precision time base is required
  - Quartz: 1 error in $10^8$-$10^9$
  - Cesium: 1 error in $10^{13}$
- The frequency being inversely proportional to the time, a counter can be used up to
  - 1 ns => 1GHz
- At higher frequencies:
  - Change of frequency (heterodyning)
  - Transfer oscillator
  - Frequency division
Frequency changesements

- Need of a non linear element
  - diode
  - transistor
- The non linearity will lead to intermodulation products
- Frequency mixing

\[ I = I_s \left( e^{\alpha U} - 1 \right) = I_s \left[ \alpha U + \frac{(\alpha U)^2}{2!} + \frac{(\alpha U)^3}{3!} + \ldots \right] \]
Mixer Principle

\[ I(V) = I_s (e^{\alpha V} - 1) \quad \alpha = \frac{q}{nKT} \]

\[ v(t) = V_1 \sin \omega_1 t + V_2 \sin \omega_2 t \]

\[ i(t) = I_s \left[ \alpha (V_1 \sin \omega_1 t + V_2 \sin \omega_2 t) + \frac{\alpha^2}{2} (V_1 \sin \omega_1 t + V_2 \sin \omega_2 t)^2 + \ldots \right] \]

\[ = I_s \left[ V_1 \sin \omega_1 t + V_2 \sin \omega_2 t \right] + \frac{\alpha^2}{2} \left[ V_1^2 \sin^2 \omega_1 t + V_2^2 \sin^2 \omega_2 t + V_1 V_2 \cos(\omega_1 - \omega_2) t - V_1 V_2 \cos(\omega_1 + \omega_2) t + \ldots \right] \]
Mixer Principle

Heterodyne counter

- Sign ambiguity
- 2 measurements are necessary
Transfer oscillator

\[ f_x = nf_{LO1} \]
\[ f_x = (n-1)f_{LO2} \]
\[ n = \frac{f_{LO2}}{(f_{LO1} - f_{LO2})} \]

"Modern" frequency counter

\[ f_C = f_x - Nf_0 \]
Spectrum analyser: the principle

filter response

spectrum of the signal

Frequency

tunable band-pass filter

signal to be measured

current ramp

variable current source

detector

y

x
Spectrum analyser: the principle

- Filter response
- Spectrum of the signal
- Tunable band-pass filter
- Signal to be measured
- Current ramp
- Variable current source
- Detector
- Y
- X

A. Skrivervik, LEMA
Spectrum analyser: the principle

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A. Skrivervik, LEMA
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EPFL
Spectrum analyser: the principle

Filter response

Spectrum of the signal

Frequency

Tunable band-pass filter

Signal to be measured

Current ramp

Variable current source

Detector

Y

X

The YIG filter
## Typical Yig filter response

### 500MHz to 18GHz YIG Filters

#### STANDARD OCTAVE BANDS

<table>
<thead>
<tr>
<th>Type</th>
<th>Minimum Model</th>
<th>Frequency Range (GHz)</th>
<th>Insertion Loss (dB)</th>
<th>Bandwidth (MHz)</th>
<th>Off Resonance Spurious Minimum (dB)</th>
<th>Combined Passband &amp; Spur Max (dB)</th>
<th>Freq Drift 0°C to 50°C (MHz)</th>
<th>Off Resonance Isolation Minimum (dB)</th>
<th>Dimensions Inside (INCHES)</th>
<th>Weight (oz)</th>
<th>Freq Tracking Between Channels (MHz)</th>
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#### STANDARD MULTI-OCTAVE BANDS

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Typical Yig filter response

Solution: fixed filter
Spectrum analyser: problems

- ambiguity
- spurious images
Ambiguity

Ambiguity : solution

\[ f_{if} = 2 \text{ [GHz]} \]

\[ f_{if} + f_2 \]

\[ f_{ff} + f_2 \]

\[ f_{ff} - f_2 \]

\[ f_{ff} \]

\[ f_{ff} f_2 \]

\[ 2f_{ff} f_2 \]

\[ 3f_{ff} f_2 \]

\[ 4f_{ff} f_2 \]
ambiguity

allowed band

spurious responses
Spurious responses: pre-filter is not a solution

Spurious responses allowed band

VCO

voltage ramp generator

\[ f_{\text{S}} \rightarrow f_{\text{S}} \rightarrow f_{\text{IF}} \rightarrow f_{\text{IF}} \rightarrow y \]

\[ f_{\text{L0}} \]

\[ x \]

\[ f_{\text{f}}=2 \text{ [GHz]} \]

allowed band

\[ f_{0} \]

\[ f_{0} \pm f_{\text{IF}} \]

\[ 2f_{0} \pm f_{\text{IF}} \]

\[ 3f_{0} \pm f_{\text{IF}} \]

\[ 4f_{0} \pm f_{\text{IF}} \]
Spurious responses: pre-selector is the solution

YIG filter

VCO

voltage ramp generator

\( f_s \) \( f_{10} \) \( f_{if} \) \( f_s \)

\( x \) \( y \)

Spurious responses

allowed band

\( f_{0+if} \) \( 2f_{0+if} \) \( 3f_{0+if} \) \( 4f_{0+if} \) \( f_{0-if} \) \( 2f_{0-if} \) \( 3f_{0-if} \) \( 4f_{0-if} \)